# Rešavanje kola metodom potencijala čvorova

- Primer kola sa $n_C = 4$ čvora
- Jedan čvor se izabere kao referentni (ima nulti potencijal), a ostalim čvorovima se pridruže potencijali $V_1$, $V_2$ i $V_3$
- Napiše se sistem od $n_C - 1 = 3$ jednačine, sa 3 nepoznate:

$$G_{11}V_1 + G_{12}V_2 + G_{13}V_3 = I^{(1)}$$
$$G_{21}V_1 + G_{22}V_2 + G_{23}V_3 = I^{(2)}$$
$$G_{31}V_1 + G_{32}V_2 + G_{33}V_3 = I^{(3)}$$

pri čemu su:

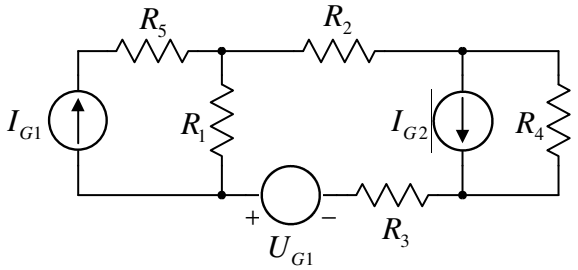
$V_1$, $V_2$ i $V_3$ - potencijali čvorova u kolu;

$G_{ii}$ ($i$=1,2,3) – zbir provodnosti grana koje se stiču u čvoru $i$; uvek ima pozitivan predznak;

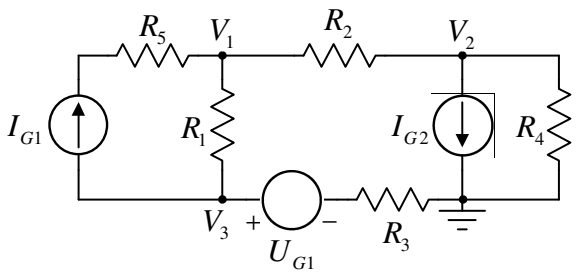
$G_{ij}$ ($i \neq j$) ($i$,$j$=1,2,3) – zbir provodnosti grana koje direktno povezuju čvorove $i$, $j$; uvek ima negativan predznak;

$I^{(i)}$ - suma struja strujnih generatora koji se stiču u čvoru $i$ (sa pozitivnim predznakom ako je referentni smer struje usmeren ka čvoru, a u suprotnom sa negativnim predznakom), plus suma napona naponskih generatora (čije se grane stiču u čvoru $i$; reč je o granama kola u kojima se nalazi redna veza idealnog naponskog generatora i otpornika) podeljenih sa otpornošću redno vezanom za te generatore (sa pozitivnim predznakom ako je „+“ u referentnom smeru naponskog generatora usmeren ka čvoru, a u suprotnom sa negativnim predznakom).

**19.** Za kolo sa slike su poznati parametri $U_{G1}$, $I_{G1}$, $I_{G2}$, $R_1$, $R_2$, $R_3$ i $R_4$. Napisati sistem jednačina po metodu potencijala čvorova za ovo kolo.



**Rešenje:**

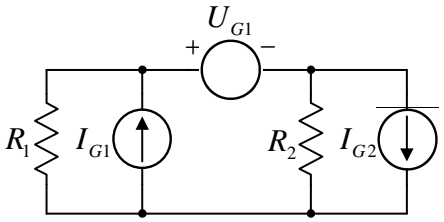



Kolo ima $n_C = 4$ čvora, pa će sistem jednačina imati $n_C - 1 = 3$ jednačine.

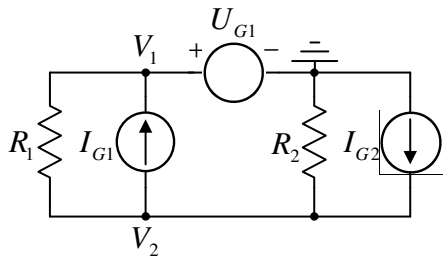
Referentni čvor se u ovom kolu <u>proizvoljno</u> bira.

$$\left.\begin{aligned} G_{11}V_1 + G_{12}V_2 + G_{13}V_3 &= I^{(1)} \\ G_{21}V_1 + G_{22}V_2 + G_{23}V_3 &= I^{(2)} \\ G_{31}V_1 + G_{32}V_2 + G_{33}V_3 &= I^{(3)} \end{aligned}\right\} \quad \Rightarrow \quad \left.\begin{aligned} \left(\frac{1}{R_1}+\frac{1}{R_2}+0\right)V_1 - \frac{1}{R_2}V_2 - \left(\frac{1}{R_1}+0\right)V_3 &= I_{G1} \\ -\frac{1}{R_2}V_1 + \left(\frac{1}{R_2}+\frac{1}{R_4}+0\right)V_2 - 0\cdot V_3 &= -I_{G2} \\ -\left(\frac{1}{R_1}+0\right)V_1 - 0\cdot V_2 + \left(\frac{1}{R_1}+\frac{1}{0+R_3}+0\right)V_3 &= -I_{G1} + \frac{U_{G1}}{R_3} \end{aligned}\right\}$$

**20.** Za kolo sa slike su poznati parametri $U_{G1}$, $I_{G1}$, $I_{G2}$, $R_1$ i $R_2$. Napisati sistem jednačina po metodu potencijala čvorova za ovo kolo.




**Rešenje:**



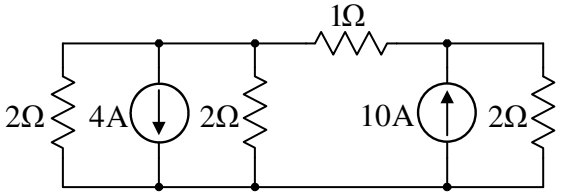

Kolo ima $n_C = 3$ čvora, pa će sistem jednačina imati $n_C - 1 = 2$ jednačine.

Za referentni čvor u ovom kolu <u>se mora</u> izabrati negativni pol idealnog naponskog generatora $U_{G1}$ (jer je $U_{G1}$ direktno povezan između dva čvora kola).

Umesto sistema jednačina: $\left.\begin{aligned} G_{11}V_1 + G_{12}V_2 &= I^{(1)} \\ G_{21}V_1 + G_{22}V_2 &= I^{(2)} \end{aligned}\right\}$, imaćemo uprošćen sistem:

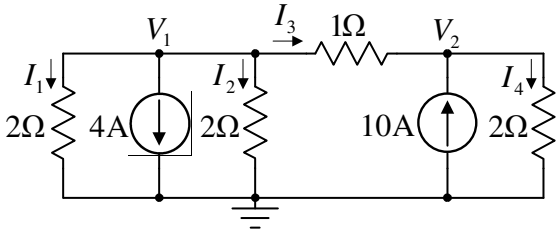
$$\left.\begin{aligned} V_1 &= U_{G1} \\ G_{21}V_1 + G_{22}V_2 &= I^{(2)} \end{aligned}\right\} \quad \Rightarrow \quad \left.\begin{aligned} V_1 &= U_{G1} \\ -\left(\frac{1}{R_1}+0\right)V_1 + \left(\frac{1}{R_1}+\frac{1}{R_2}+0+0\right)V_2 &= -I_{G1}+I_{G2} \end{aligned}\right\}$$

**21.** Primenom metode potencijala čvorova izračunati potencijale svih čvorova i struje kroz sve otpornike u kolu sa slike.




**Rešenje:**

$$\left.\begin{aligned} G_{11}V_1 + G_{12}V_2 &= I^{(1)} \\ G_{21}V_1 + G_{22}V_2 &= I^{(2)} \end{aligned}\right\}$$



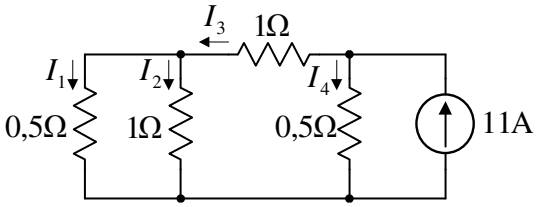

$$\left.\begin{aligned} \left(\frac{1}{2\Omega}+\frac{1}{2\Omega}+\frac{1}{1\Omega}+0\right)V_1 - \frac{1}{1\Omega}V_2 &= -4\text{A} \\ -\frac{1}{1\Omega}V_1 + \left(\frac{1}{1\Omega}+\frac{1}{2\Omega}+0\right)V_2 &= 10\text{A} \end{aligned}\right\}$$

$$\left.\begin{aligned} 2V_1 - V_2 &= -4 \\ -V_1 + \frac{3}{2}V_2 &= 10 \;/\cdot 2 \end{aligned}\right\} \quad \Rightarrow \quad \left.\begin{aligned} 2V_1 - V_2 &= -4 \\ -2V_1 + 3V_2 &= 20 \end{aligned}\right\} + \quad \Rightarrow \quad 2V_2 = 16 \quad \Rightarrow \quad \boxed{V_2 = 8\text{V}}, \quad \boxed{V_1 = 2\text{V}}$$

$$I_1 = \frac{V_1 - 0}{2\Omega} \quad \Rightarrow \quad \boxed{I_1 = 1\text{A}} \qquad I_2 = \frac{V_1 - 0}{2\Omega} \quad \Rightarrow \quad \boxed{I_2 = 1\text{A}}$$

$$I_3 = \frac{V_1 - V_2}{1\Omega} \quad \Rightarrow \quad \boxed{I_3 = -6\text{A}} \qquad I_4 = \frac{V_2 - 0}{2\Omega} \quad \Rightarrow \quad \boxed{I_4 = 4\text{A}}$$

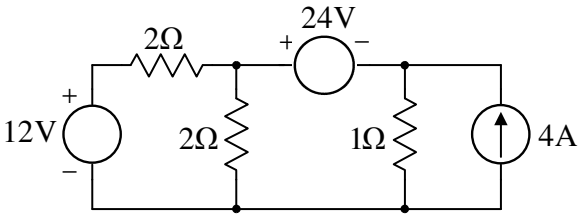
**22.** (Zadatak za vežbu) Primenom metode potencijala čvorova izračunati struje $I_1$, $I_2$, $I_3$ i $I_4$ u kolu sa slike.



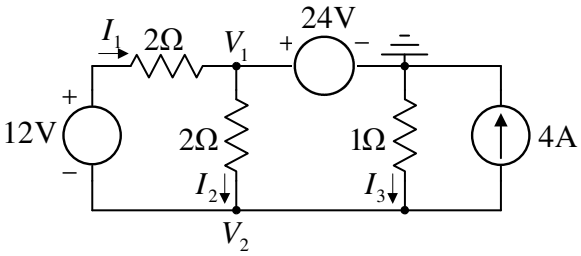

**Rešenje:**

$\boxed{I_1 = 2\text{A}}$, $\boxed{I_2 = 1\text{A}}$, $\boxed{I_3 = 3\text{A}}$, $\boxed{I_4 = 8\text{A}}$

**23.** Primenom metode potencijala čvorova izračunati potencijale svih čvorova i struje kroz sve otpornike u kolu sa slike.




**Rešenje:**

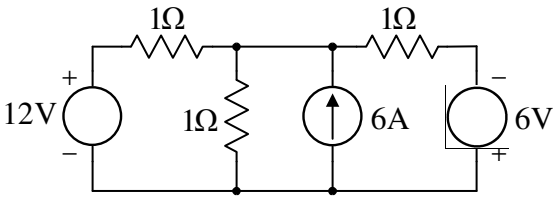



$$\left.\begin{array}{l} V_1 = 24\text{V} \\ -\left(\dfrac{1}{2\Omega}+\dfrac{1}{2\Omega+0}\right)V_1+\left(\dfrac{1}{2\Omega+0}+\dfrac{1}{2\Omega}+\dfrac{1}{1\Omega}+0\right)V_2 = -4\text{A}-\dfrac{12\text{V}}{2\Omega} \end{array}\right\} \Rightarrow \left.\begin{array}{l} V_1 = 24\text{V} \\ -V_1+2V_2 = -10 \end{array}\right\}$$

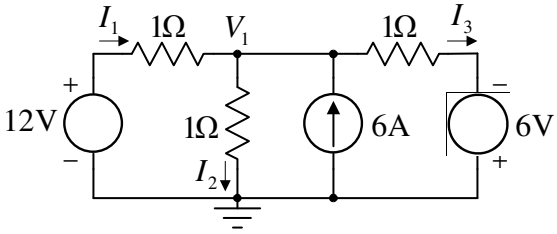
$\boxed{V_2 = 7\text{V}}$, $\boxed{V_1 = 24\text{V}}$

$I_1 = \dfrac{(V_2+12\text{V})-V_1}{2\Omega} \Rightarrow \boxed{I_1 = -2{,}5\text{A}}$, $I_2 = \dfrac{V_1-V_2}{2\Omega} \Rightarrow \boxed{I_2 = 8{,}5\text{A}}$, $I_3 = \dfrac{0-V_2}{1\Omega} \Rightarrow \boxed{I_3 = -7\text{A}}$

**24.** Primenom metode potencijala čvorova izračunati potencijale svih čvorova i struje kroz sve otpornike u kolu sa slike.




**Rešenje:**




$G_{11}V_1 = I^{(1)}$

$$\left(\frac{1}{1\Omega+0}+\frac{1}{1\Omega}+0+\frac{1}{1\Omega+0}\right)V_1 = 6\text{A}+\frac{12\text{V}}{1\Omega}-\frac{6\text{V}}{1\Omega}$$

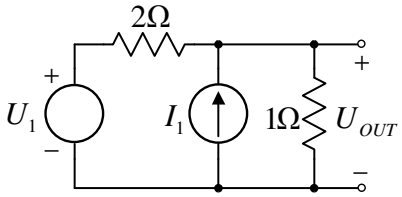
$3V_1 = 12 \quad \Rightarrow \quad \boxed{V_1 = 4\text{V}}$

$$I_1 = \frac{12\text{V}-V_1}{1\Omega} \quad \Rightarrow \quad \boxed{I_1 = 8\text{A}}, \qquad I_2 = \frac{V_1-0}{1\Omega} \quad \Rightarrow \quad \boxed{I_2 = 4\text{A}},$$

$$I_3 = \frac{V_1-(-6\text{V})}{1\Omega} \quad \Rightarrow \quad \boxed{I_3 = 10\text{A}}$$

# Rešavanje kola primenom metode superpozicije

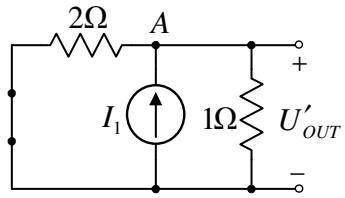
**25.** Primenom metode superpozicije odrediti napon $U_{OUT}$ u kolu sa slike. Smatrati da su $U_1$ i $I_1$ poznati parametri.




**Rešenje:**

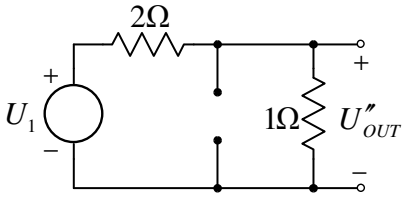
$U_{OUT} = U'_{OUT} + U''_{OUT}$

$U'_{OUT} = U_{OUT}(U_1 = 0, I_1 \neq 0)$ $\qquad$ $U''_{OUT} = U_{OUT}(U_1 \neq 0, I_1 = 0)$




Čvor $A$: $\quad I_1 - \dfrac{U'_{OUT}}{1\Omega} - \dfrac{U'_{OUT}}{2\Omega} = 0$

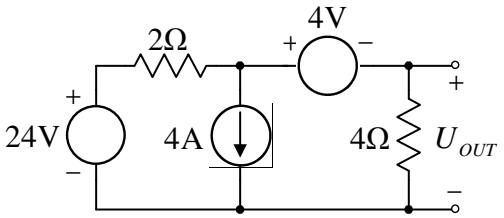
$U'_{OUT} = \dfrac{2}{3} I_1$



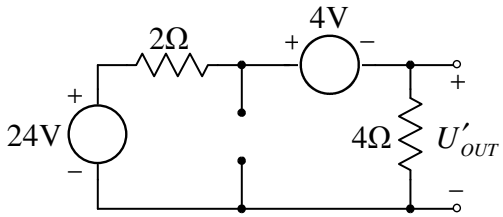

$U''_{OUT} = \dfrac{1\Omega}{1\Omega + 2\Omega} U_1 = \dfrac{1}{3} U_1$

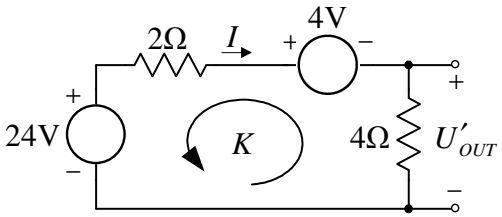
$$\boxed{U_{OUT} = \frac{2}{3} I_1 + \frac{1}{3} U_1}$$

**26.** Primenom metode superpozicije odrediti napon $U_{OUT}$ u kolu sa slike.




**<u>Rešenje:</u>**

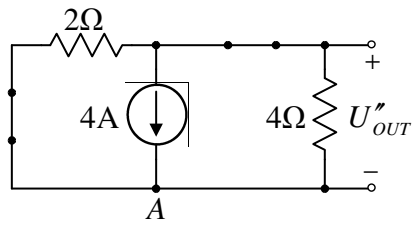



Kontura $K$: $\quad I \cdot 4\Omega + 4\text{V} + I \cdot 2\Omega - 24\text{V} = 0$

$I \cdot 6\Omega = 20\text{V} \quad \Rightarrow \quad I = \dfrac{10}{3}\text{A}$

$U'_{OUT} = I \cdot 4\Omega = \dfrac{40}{3}\text{V}$

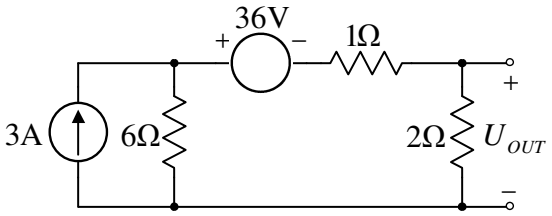



Čvor $A$: $\quad 4\text{A} + \dfrac{U''_{OUT}}{2\Omega} + \dfrac{U''_{OUT}}{4\Omega} = 0$

$$\frac{3}{4}U''_{OUT} = -4\text{V} \quad \Rightarrow \quad U''_{OUT} = -\frac{16}{3}\text{V}$$

$$U_{OUT} = U'_{OUT} + U''_{OUT} \quad \Rightarrow \quad \boxed{U_{OUT} = 8\text{V}}$$

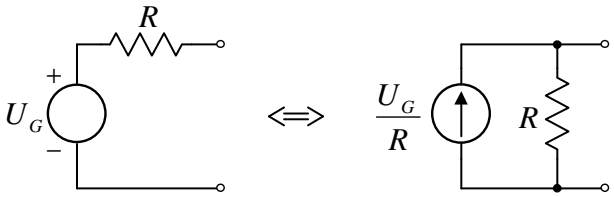
**27.** (Zadatak za vežbu) Primenom metode superpozicije odrediti napon $U_{OUT}$ u kolu sa slike.



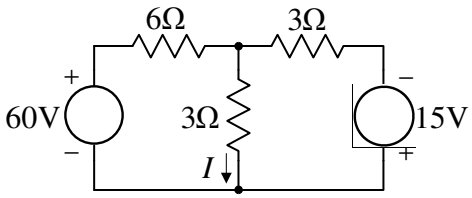

**Rešenje:**

$\boxed{U_{OUT} = -4\text{V}}$

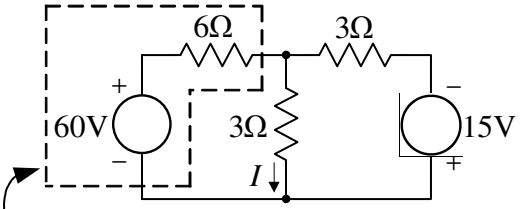
## Rešavanje kola primenom metode transformacije izvora

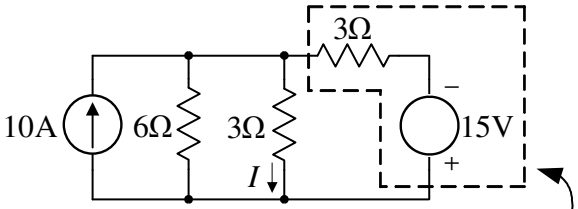


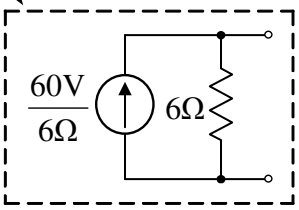

**28.** Primenom metode transformacije izvora odrediti struju $I$ u kolu sa slike.

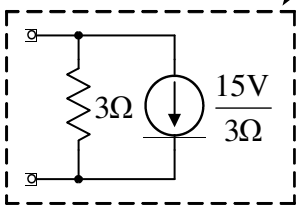


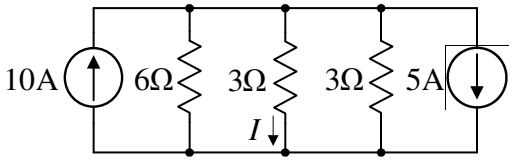
**Rešenje:**

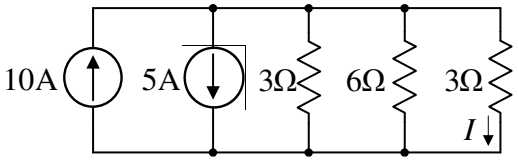


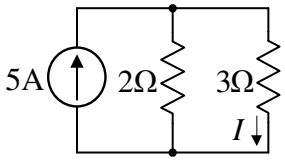



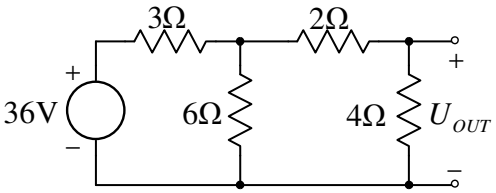

$$I = \frac{2\Omega}{2\Omega + 3\Omega} \cdot 5\text{A} \quad \Rightarrow \quad \boxed{I = 2\text{A}}$$

# Rešavanje kola primenom Tevenenove i Nortonove teoreme

**29.** Za kolo sa slike odrediti napon $U_{OUT}$ primenom:

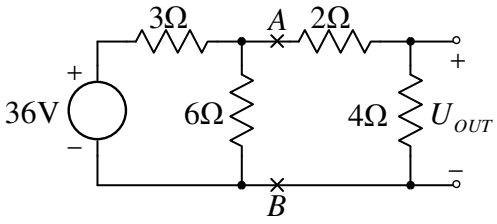
**a)** Tevenenove teoreme;

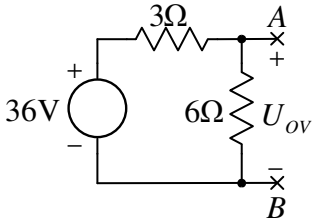
**b)** Nortonove teoreme.

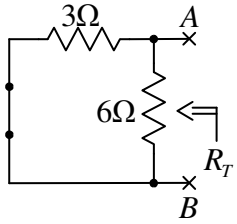


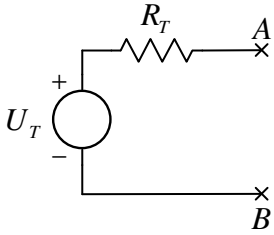

**Rešenje:**

**a)**



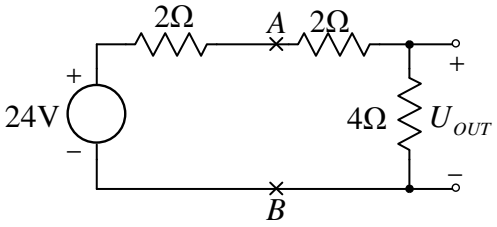



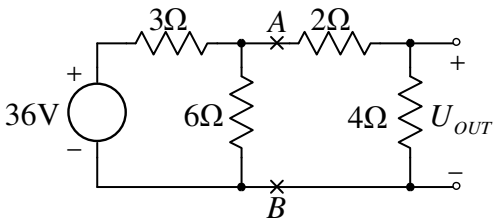

$$U_T = U_{OV} = \frac{6\Omega}{6\Omega + 3\Omega} \cdot 36\text{V} = 24\text{V}$$

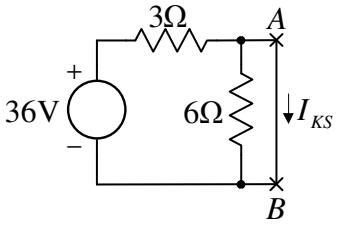
$$R_T = 6\Omega \parallel 3\Omega = 2\Omega$$

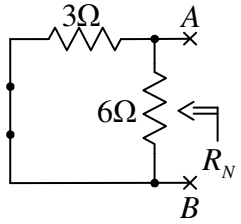


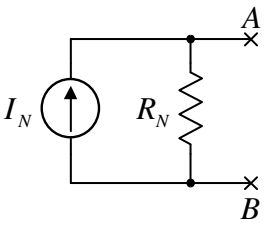

$$U_{OUT} = \frac{4\Omega}{4\Omega + (2\Omega + 2\Omega)} \cdot 24\text{V} \quad \Rightarrow \quad \boxed{U_{OUT} = 12\text{V}}$$

**b)**



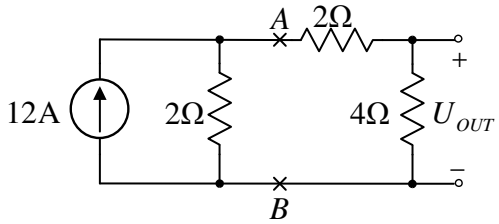



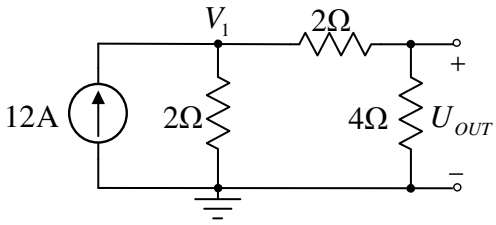




$I_{KS} \cdot 3\Omega - 36\text{V} = 0 \quad \Rightarrow \quad I_N = I_{KS} = 12\text{A}$

$R_N = 6\Omega \parallel 3\Omega = 2\Omega$

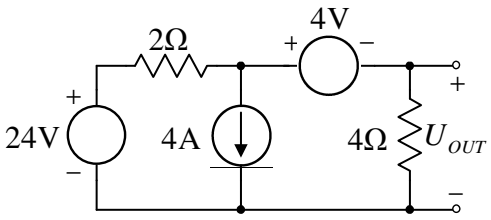


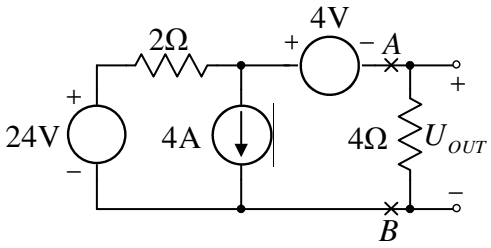

MPČ: $\left(\frac{1}{2\Omega}+\frac{1}{6\Omega}\right)V_1 = 12\text{A} \quad \Rightarrow \quad V_1 = 18\text{V}$

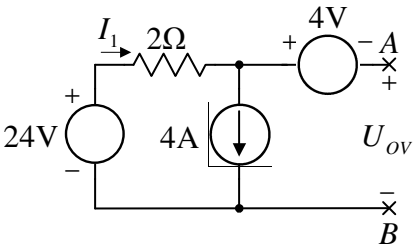
$U_{OUT} = \frac{4\Omega}{4\Omega + 2\Omega} \cdot V_1 \quad \Rightarrow \quad \boxed{U_{OUT} = 12\text{V}}$

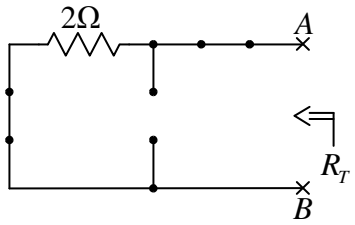
**30.** Za kolo sa slike odrediti napon $U_{OUT}$ primenom Tevenenove teoreme.

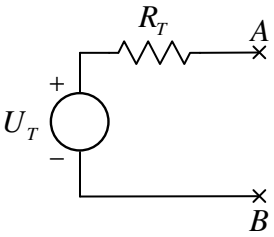


**Rešenje:**



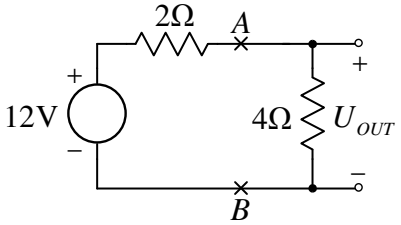



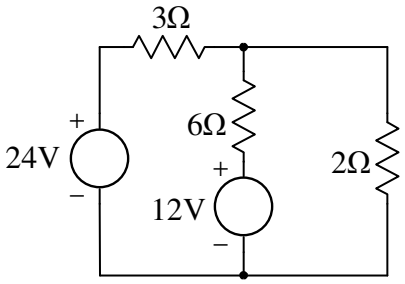

$I_1 = 4\text{A}$ $\qquad\qquad$ $R_T = 2\Omega$

$U_T = U_{OV} = 24\text{V} - I_1 \cdot 2\Omega - 4\text{V} = 12\text{V}$

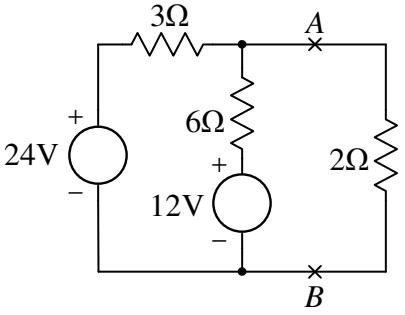


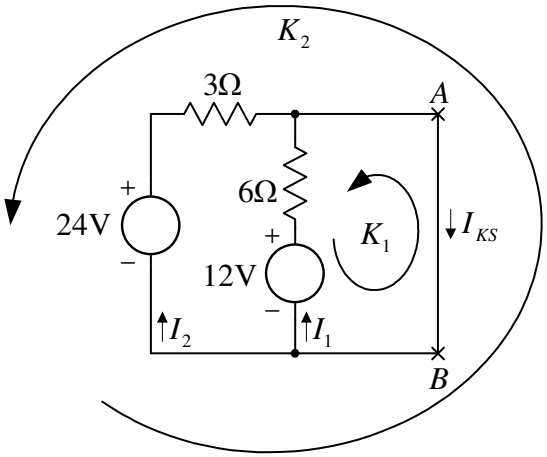

$$U_{OUT} = \frac{4\Omega}{4\Omega + 2\Omega} \cdot 12\text{V} \quad \Rightarrow \quad \boxed{U_{OUT} = 8\text{V}}$$

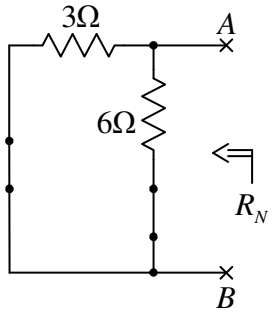
**31.** Za kolo sa slike odrediti snagu koja se disipira na otporniku otpornosti $2\Omega$ primenom Nortonove teoreme.



**Rešenje:**



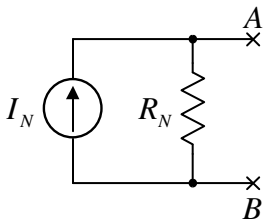



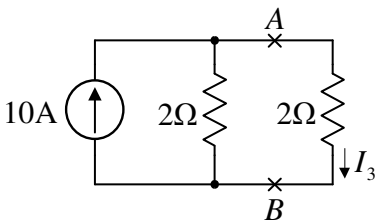

Kontura $K_1$: $I_1 \cdot 6\Omega - 12\text{V} = 0 \quad \Rightarrow \quad I_1 = 2\text{A}$

$R_N = 6\Omega \parallel 3\Omega = 2\Omega$

Kontura $K_2$: $I_2 \cdot 3\Omega - 24\text{V} = 0 \quad \Rightarrow \quad I_2 = 8\text{A}$

$I_N = I_{KS} = I_1 + I_2 = 10\text{A}$



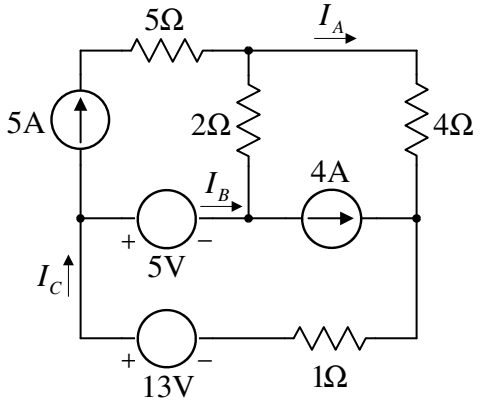




$$I_3 = \frac{2\Omega}{2\Omega + 2\Omega} \cdot 10\text{A} \quad \Rightarrow \quad I_3 = 5\text{A}$$

$$P_{2\Omega} = I_3^2 \cdot 2\Omega \quad \Rightarrow \quad \boxed{P_{2\Omega} = 50\text{W}}$$

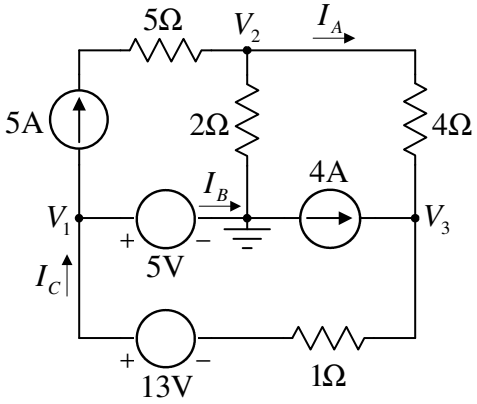
**32. a)** Primenom metode potencijala čvorova izračunati potencijale svih čvorova u kolu sa slike, kao i struje $I_A$, $I_B$ i $I_C$.

**b)** Korišćenjem rezultata iz prethodne tačke, za svaki od generatora izračunati snagu koju predaje.




**Rešenje:**

**a)**




$$\left.\begin{aligned} &V_1 = 5\text{V} \\ &-0\cdot V_1 + \left(\frac{1}{2\Omega}+\frac{1}{4\Omega}+0\right)V_2 - \frac{1}{4\Omega}\cdot V_3 = 5\text{A} \\ &-\frac{1}{1\Omega}\cdot V_1 - \frac{1}{4\Omega}\cdot V_2 + \left(\frac{1}{4\Omega}+\frac{1}{1\Omega}+0\right)V_3 = 4\text{A} - \frac{13\text{V}}{1\Omega} \end{aligned}\right\}$$

$$\left.\begin{aligned} &V_1 = 5\text{V} \\ &\frac{3}{4}V_2 - \frac{1}{4}V_3 = 5 \quad\Rightarrow\quad V_3 = 3V_2 - 20 \\ &-V_1 - \frac{1}{4}V_2 + \frac{5}{4}V_3 = -9 \end{aligned}\right\}$$

$$\left.\begin{aligned} &V_1 = 5\text{V} \\ &V_3 = 3V_2 - 20 \\ &-5 - \frac{1}{4}V_2 + \frac{15}{4}V_2 - 25 = -9 \quad\Rightarrow\quad V_2 = 6\text{V} \end{aligned}\right\}$$

$$\boxed{V_2 = 6\text{V}}, \qquad \boxed{V_3 = -2\text{V}}, \qquad \boxed{V_1 = 5\text{V}}$$

$$I_A = \frac{V_2 - V_3}{4\Omega} \Rightarrow \boxed{I_A = 2\text{A}}, \qquad I_C = \frac{V_3 - (V_1 - 13\text{V})}{1\Omega} \Rightarrow \boxed{I_C = 6\text{A}}, \qquad I_B = I_C - 5\text{A} \Rightarrow \boxed{I_B = 1\text{A}}$$

**b)** $P_{5\text{V}} = (-I_B)\cdot 5\text{V} = (-1\text{A})\cdot 5\text{V} \quad\Rightarrow\quad \boxed{P_{5\text{V}} = -5\text{W}}$

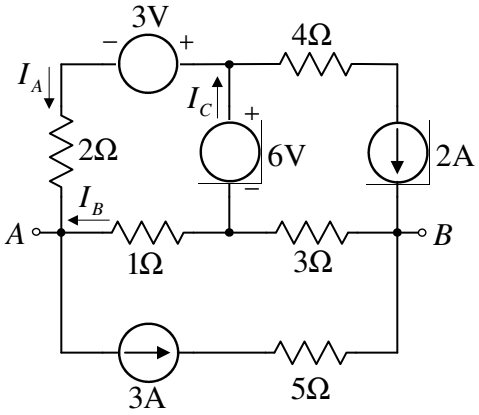
$P_{13\text{V}} = I_C \cdot 13\text{V} = 6\text{A}\cdot 13\text{V} \quad\Rightarrow\quad \boxed{P_{13\text{V}} = 78\text{W}}$

$$P_{4\mathrm{A}} = 4\mathrm{A}\cdot(\mathrm{V}_3 - 0) = 4\mathrm{A}\cdot(-2\mathrm{V}) \quad \Rightarrow \quad \boxed{P_{4\mathrm{A}} = -8\mathrm{W}}$$

$$P_{5\mathrm{A}} = 5\mathrm{A}\cdot(\mathrm{V}_2 + 5\mathrm{A}\cdot 5\Omega - V_1) = 5\mathrm{A}\cdot 26\mathrm{V} \quad \Rightarrow \quad \boxed{P_{5\mathrm{A}} = 130\mathrm{W}}$$

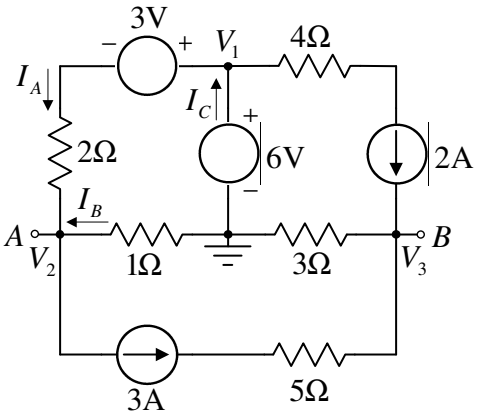
**33. a)** Primenom metode potencijala čvorova izračunati potencijale svih čvorova u kolu sa slike, kao i struje $I_A$, $I_B$ i $I_C$.

**b)** Korišćenjem rezultata iz prethodne tačke, odrediti parametre ekvivalentnog Tevenenovog generatora između tačaka A i B.




**Rešenje:**

**a)**

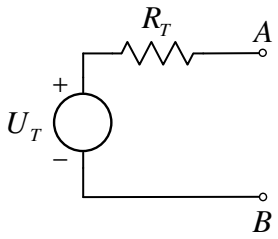



$$\left.\begin{aligned} &V_1 = 6\mathrm{V} \\ &-\frac{1}{2\Omega}\cdot V_1 + \left(\frac{1}{2\Omega} + \frac{1}{1\Omega} + 0\right)V_2 - 0\cdot V_3 = -3\mathrm{A} - \frac{3\mathrm{V}}{2\Omega} \\ &-0\cdot V_1 - 0\cdot V_2 + \left(\frac{1}{3\Omega} + 0 + 0\right)V_3 = 3\mathrm{A} + 2\mathrm{A} \end{aligned}\right\}$$

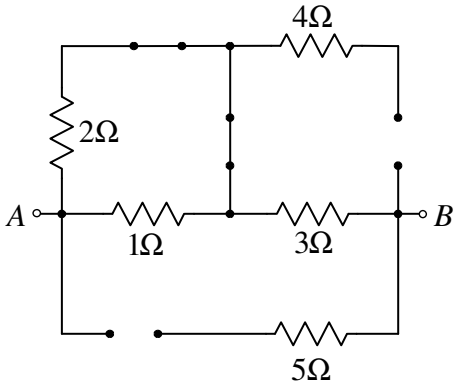
$$\left.\begin{aligned} &V_1 = 6\mathrm{V} \\ &-\frac{1}{2}V_1 + \frac{3}{2}V_2 = -\frac{9}{2} \quad \Rightarrow \quad V_2 = \frac{1}{3}V_1 - 3 \\ &\frac{1}{3}V_3 = 5 \end{aligned}\right\} \quad \Rightarrow \quad \boxed{V_1 = 6\mathrm{V}}, \quad \boxed{V_3 = 15\mathrm{V}}, \quad \boxed{V_2 = -1\mathrm{V}}$$

$$I_A = \frac{(V_1 - 3\text{V}) - V_2}{2\Omega} \Rightarrow \boxed{I_A = 2\text{A}}, \qquad I_B = \frac{0 - V_2}{1\Omega} \Rightarrow \boxed{I_B = 1\text{A}}, \qquad I_C = I_A + 2\text{A} \Rightarrow \boxed{I_C = 4\text{A}}$$

**b)**




$$U_T = V_A - V_B = V_2 - V_3 \quad \Rightarrow \quad \boxed{U_T = -16\text{V}}$$




$$R_T = R_{AB} = 3\Omega + 1\Omega \parallel 2\Omega \quad \Rightarrow \quad \boxed{R_T = 3{,}67\Omega}$$